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# New Nonequilibrium Turbulence Model for Calculating Flows over Airfoils

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## Abstract

A NONEQUILIBRIUM turbulence closure model, patterned after the Johnson-King model, has been developed for computing two-dimensional turbulent flows about complex airfoil shapes. The influence of history effects in the wake region of the boundary layer is described by an ordinary differential equation developed from the turbulent kinetic energy equation. The performance of the present model has been evaluated by computing the flow around three airfoils using the Reynolds time-averaged Navier-Stokes equations. These equations were solved using an implicit, upwind, finite-volume scheme. Excellent agreement between numerical and experimental results was obtained for both attached and separated turbulent flows about the NACA 0012 airfoil, the RAE 2822 airfoil, and the Integrated Technology A 153W airfoil.

## Contents

The majority of turbulent flow calculations are based on the Reynolds time-averaged equations. These equations can be obtained by using Favre's mass-weighted averaging. Closure of the Reynolds time-averaged equations requires the modeling of the turbulent stresses and heat-flux terms. In most turbulence models, the turbulent stresses are determined from the Boussinesq approximation:

$$\overline{-\rho u_i' u_j'} = \mu_t\left[\left(\frac{\partial \tilde{u}_i}{\partial x_j}+\frac{\partial \tilde{u}_j}{\partial x_i}\right)-\frac{2}{3}\delta_{ij}\frac{\partial \tilde{u}_k}{\partial x_k}\right]+\frac{2}{3}\delta_{ij}\bar{\rho}\bar{k} \qquad (1)$$

where $\bar{k}$ is the mean turbulent kinetic energy per unit mass. The apparent heat flux is related to the turbulent viscosity $\mu_t$, the mean flow variables, and the turbulent Prandtl number $Pr_t$.

It is obvious that the success of a prediction method for turbulent flows depends to a large extent on the choice of the turbulence model. Algebraic (equilibrium) models are generally inadequate for nonequilibrium turbulent flows and flows that are not self-preserving. For reliable computations of these flows, the so-called history effects of convection and diffusion of turbulent energy must be taken into consideration. Based on the argument that the turbulent shear stress is closely related to the turbulent kinetic energy, an effort was undertaken to develop a new two-layer turbulence model from a variation of the turbulent energy equation. In order to do this, it was first appreciated that history effects are likely to be much more important in the outer layer than in the inner layer. This is due to the fact that the outer layer is dominated by large turbulent eddies with considerably longer lifetimes. As a result, an ordinary differential equation, derived from the turbulent kinetic energy equation, is used in the present model to describe the effects of convection and diffusion of turbulent shear stress in this layer. Although it bears a strong resemblance to the Johnson-King model,[1] the present nonequilibrium turbulence model is unique. In addition, it is much easier to incorporate into a Navier-Stokes solver than the Johnson-King model since it does not involve any iteration.

Because mass-weighted averaging accounts for much of the effects of compressibility on turbulence at moderate Mach numbers, the equation for mean turbulent kinetic energy in an incompressible flow is assumed to hold for compressible turbulent flows. Thus, the starting equation for the development of the present turbulence model is the mean turbulent kinetic energy equation for two-dimensional incompressible flow outside the viscous sublayer. In order to adequately capture history effects in the outer layer with minimum effort, the above equation is written along a particular path resulting in an ordinary differential equation. It was found that there is a path in the outer region along which the function $y\tau$ ($\tau=\overline{u'v'}$) is maximum. Along this path, the aforementioned equation for turbulent kinetic energy can be written as

$$\tilde{u}_m y_m \frac{\partial \tau_m}{\partial x} - \tilde{v}_m \tau_m - a_1 y_m\left[\tau_m\left(\frac{\partial \tilde{u}}{\partial y}\right)_m + \left(\frac{\partial D_s}{\partial y}\right) + \frac{\tau_m^{3/2}}{L}\right] = 0 \qquad (2)$$

where the term involving $\partial D_s/\partial y$ represents the diffusion of turbulence, $\tau = a_1\bar{k}$, and the dissipation length scale $L = \tau^{3/2}/\epsilon = 0.09\delta$. Here and in what follows, the subscript "$m$" denotes values where the value of $y\tau$ is maximum. In Eq. (2), $L\partial\tilde{u}/\partial y$ can be interpreted as the square root of the ratio of turbulent shear stress to density that would result if convection and diffusion effects were negligible. This term is then replaced by $(\overline{-u'v'_{\text{eq}}})^{1/2}$, which is assumed to be determined from an equilibrium eddy-viscosity model. Owing to its ease in computation, the Baldwin-Lomax model is chosen here as the required equilibrium model in the outer layer. The diffusion term in Eq. (2) is modeled in a manner similar to that used by Johnson-King[1] and is given in Ref. 2. Defining $g = (\overline{-u'v'})_m^{-1/2}$, Eq. (2) is finally rearranged as

$$\frac{\mathrm{d}g}{\mathrm{d}x} = \frac{a_1}{2\tilde{u}_m L} - g\left[\frac{a_1}{2\tilde{u}_m L g_{\text{eq}}} + \frac{\tilde{v}_m}{2\tilde{u}_m y_m} - \frac{C_{\text{model}}\tau_{\max}^{1/2}}{2\tilde{u}_m(0.7\delta - y_{\max})}\frac{y_{\max}}{y_m}\right] \qquad (3)$$

Here $C_{\text{model}}$ is a modeling constant taken to have a value of 0.3 and the subscript "max" refers to the location where $\overline{-u'v'}$ is maximum. Equation (3) can be easily solved if all of the terms on the right-hand side except $g$ and $g_{\text{eq}}$ are determined at the previous streamwise location. The calculation of the eddy viscosity is started with the Baldwin-Lomax model and then switched to the present model. With $\tilde{\gamma}$ as the Klebanoff intermittency function, the eddy viscosity in the outer layer is then given by

$$\nu_{t_o} = \frac{1}{g^2(\partial\tilde{u}/\partial y + \partial\tilde{v}/\partial x)_m}\left(\frac{\tilde{\gamma}}{\tilde{\gamma}_m}\right) \qquad (4)$$

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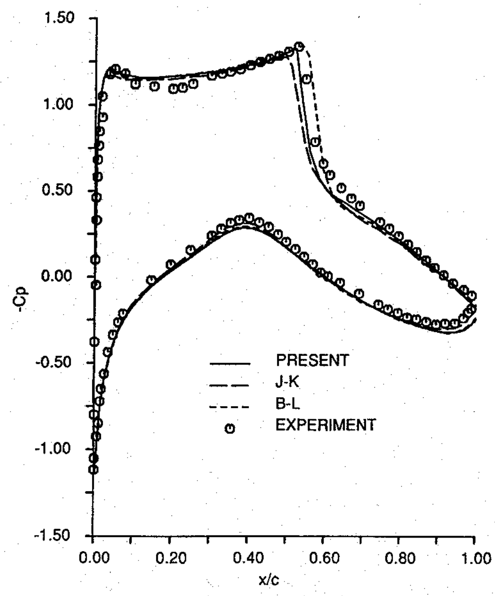


**Fig. 1 RAE 2822: effects of turbulence models on surface-pressure coefficient at $M_\infty = 0.73$, $Re = 6.5 \times 10^6$, $\alpha_{comp} = 2.80$ deg, and $\alpha_{exp} = 3.19$ deg.**

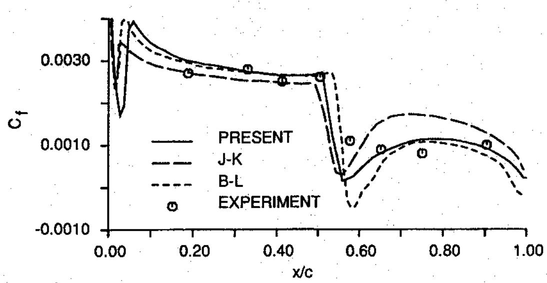


**Fig. 2 RAE 2822: skin-friction coefficient at $M_\infty = 0.73$, $Re = 6.5 \times 10^6$, $\alpha_{comp} = 2.80$ deg, and $\alpha_{exp} = 3.19$ deg.**

Since in the wall region the largest stress containing eddies have wavelengths proportional to the distance from the wall, it is assumed that the turbulence length scale is given by $l_i = D\kappa y$, where $\kappa = 0.4$ and $D$ is the Cebeci-Smith extension of the Van Driest damping function. The velocity scale of turbulence in this layer is still allowed to be affected by the history effects. Since the term $g_{eq}/g$ can be interpreted as a correction applied to the equilibrium turbulence velocity scale in the outer layer, a similar philosophical approach is taken to describe the inner turbulence velocity scale. A correction factor $F_c$ is applied to the equilibrium velocity scale $D\kappa y|\bar{\omega}|$ to introduce the nonequilibrium effects. Based on the observation that Coles velocity profiles are valid for attached and mildly separated flows, it was possible to deduce a relationship between $g_{eq}/g$ and $F_c$.[3] The kinematic eddy viscosity in the inner layer is then given as

$$\nu_{t_i} = \kappa^2 D^2 y^2 |\bar{\omega}| F_c \tag{5}$$

where

$$F_c = \frac{g_{eq}/g}{1 - f_1}, \qquad f_1 = \frac{u_\tau(\rho_w/\rho_e)^{1/2}(\tau_w/|\tau_w|)[1 - \tau_w/|\tau_w|]}{\kappa F_{max}}$$

The eddy-viscosity distribution across the entire boundary layer is then

$$\nu_t = \min(\nu_{t_o}, \nu_{t_i}) \tag{6}$$

In order to validate the new turbulence model developed in this study, the flowfields surrounding three airfoils—the NACA 0012 airfoil, the RAE 2822 airfoil, and the Integrated Technology A 153W airfoil—have been computed.[2] No adjustments were made to the turbulence model for these different test cases. The flowfields were computed using the computer code of Cox.[4] This code solves the Reynolds time-averaged Navier-Stokes equations using an implicit, upwind, finite-volume scheme. For brevity, only one flowfield about the supercritical airfoil RAE 2822 (tested extensively by Cook et al.[5]) is presented here. The flow was forced to transition at 3% of chord. An angle-of-attack correction recommended by the experimenter was applied to account for wind-tunnel wall interferences. A representative C grid employed for the airfoil calculations contained $221 \times 92$ grid cells. Figure 1 shows experimental and computed pressure distributions at a Mach number of 0.73, a Reynolds number of $6.5 \times 10^6$, and at experimental and computational angles of attack of 3.19 and 2.80 deg, respectively. In this case, there is no separation and the present result is in close agreement with experiment. A shock wave is predicted near $x/c = 0.54$ and is in excellent agreement with the experimental result. The effects of different turbulence models on surface pressure distribution are also given in this figure. The results corresponding to the Baldwin-Lomax and the Johnson-King models were taken from Ref. 6. Large differences between model predictions and experiment are indicated by the skin-friction distributions given in Fig. 2. The present model appears to give the best overall agreement in skin friction. Referring to Fig. 2, it is apparent that the Baldwin-Lomax model predicts weak separation at the shock wave and the trailing edge, whereas the Johnson-King and the present model predict no separation. The computed lift and drag coefficients using the present model are 0.822, 0.0167, respectively, while the corresponding experimental values are 0.803 and 0.0168.

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